Toward Improving the Laplacian Estimation with Novel Multipolar Concentric Ring Electrodes

Oleksandr Makeyev, *Member, IEEE*, Quan Ding, *Member, IEEE*, Steven M. Kay, *Fellow, IEEE*, and
Walter G. Besio, *Senior Member, IEEE*

Abstract— **Conventional electroencephalography with disc electrodes has major drawbacks including poor spatial resolution, selectivity and low signal-to-noise ratio that are critically limiting its use. Concentric ring electrodes are a promising alternative with potential to improve all of the aforementioned aspects significantly. In our previous work, the tripolar concentric ring electrode was successfully used in a wide range of applications demonstrating its superiority to conventional disc electrode, in particular, in accuracy of Laplacian estimation. This paper takes the first fundamental step toward further improving the Laplacian estimation of the novel multipolar concentric ring electrodes by proposing a general approach to estimation of the Laplacian for an $(n + 1)$-polar electrode with n rings using the $(4n + 1)$-point method for $n \geq 2$ that allows cancellation of all the truncation terms up to the order of $2n$. Examples of using the proposed approach to estimate the Laplacian for the cases of tripolar and, for the first time, quadripolar concentric ring electrode are presented.**

I. INTRODUCTION

Electroencephalography (EEG) is an essential tool for brain and behavioral research and is used extensively in neuroscience, cognitive science, cognitive psychology, and psychophysiology. EEG is also one of the mainstays of hospital diagnostic procedures and pre-surgical planning. Despite scalp EEG's many advantages end users struggle with its poor spatial resolution, selectivity and low signal-to-noise ratio, which are EEG's biggest drawbacks and major hindrances in its effectiveness critically limiting the research discovery and diagnosis [1]-[3].

EEG's poor spatial resolution is primarily due to (1) the blurring effects of the volume conductor with disc electrodes; and (2) EEG signals having reference electrode problems as idealized references are not available with EEG [2]. Interference on the reference electrode contaminates all other electrode signals [2]. The application of the surface Laplacian (the second spatial derivative of the potentials on the body surface) to EEG has been shown to alleviate the blurring effects enhancing the spatial resolution and selectivity [4]-[6], and reduce the reference problem.

While several methods were proposed for estimation of the surface Laplacian through interpolation of potentials on a surface and then estimating the Laplacian from an array of disc electrodes [5]-[9] concentric ring electrodes (CRE) have shown more promise. The CREs can resolve the reference electrode problems since they act like closely spaced bipolar recordings [2]. CREs are symmetrical alleviating electrode orientation problems [10]. They also act as spatial filters reducing the low spatial frequencies and increasing the spatial selectivity [10], [11]. Finally, even bipolar CRE, consisting of just two elements including a single ring and the central disc, improve the radial attenuation of the conventional disc electrode from $1/r^3$ to $1/r^4$ with higher numbers of poles having the potential to enhance radial attenuation even further [12].

Tripolar CREs (TCRE; the highest number of CRE poles currently used), consisting of three elements including the outer ring, the middle ring, and the central disc (Fig. 1, B), are distinctively different from conventional disc electrodes that have a single element (Fig. 1, A). TCREs have been shown to estimate the surface Laplacian directly through the nine-point method (NPM), an extension of the five-point method (FPM) used for bipolar CREs, and significantly better than other electrode systems including bipolar and quasi-bipolar CREs [13], [14]. Compared to EEG with conventional disc electrodes Laplacian via TCREs have been shown to have significantly better spatial selectivity (approximately 2.5 times higher), signal-to-noise ratio (approximately 3.7 times higher), and mutual information (approximately 12 times lower) [15]. TCREs also have very high common mode noise rejection providing automatic artifact attenuation, -100 dB one radius from the electrode [14]. Because of such unique capabilities TCREs have found

O. Makeyev, Q. Ding, S. M. Kay, and W. G. Besio are with the Department of Electrical, Computer, and Biomedical Engineering, University of Rhode Island, Kingston, RI 02881, USA (phone: 401-874-9208; fax: 401-782-6422; e-mail: omakeyev@ele.uri.edu, dingqqq@ele.uri.edu, kay@ele.uri.edu, besio@ele.uri.edu).

Figure 1. Conventional disc electrode (A) and tripolar concentric ring electrode (B).

numerous applications in a wide range of areas including brain-computer interface [16], seizure onset detection [17], [18] seizure attenuation using transcranial focal stimulation applied via TCREs [19]-[22], etc.

Taking a next fundamental step toward development of multipolar CREs, in this study the Laplacian is derived for a general case of $(n + 1)$-polar CRE with n rings using the $(4n + 1)$-point method for $n \geq 2$ and it is demonstrated how the accuracy of the Laplacian estimation increases with the increase of n due to elimination of higher order truncation terms. Furthermore, the Laplacian estimations for TCRE and quadripolar CRE (QCRE) are derived numerically using the proposed general case approach.

II. PRELIMINARIES AND NOTATIONS

A. Five-Point Method (Bipolar CRE)

As shown in Fig. 2 v_0 through $v_{nr,4}$ are the potentials at points p_0 through $p_{nr,4}$, respectively. To simplify the narrative, v_0 through $v_{nr,4}$ may also signify points p_0 through $p_{nr,4}$. v_0, $v_{r,1}$, $v_{r,2}$, $v_{r,3}$ and $v_{r,4}$, with a spacing of r are applied in the FPM (a bipolar CRE configuration Laplacian estimation) following Huiskamp's [23] calculation of the Laplacian. The Laplacian potentials at point p_0 are calculated using Taylor expansion:

$$\Delta v_0 = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{1}{r^2}(\sum_{i=1}^{4} v_{r,i} - 4v_0) + O(r^2) \qquad (1)$$

where $O(r^2) = \frac{r^2}{4!}(\frac{\partial^4 v}{\partial x^4} + \frac{\partial^4 v}{\partial y^4}) + \frac{r^4}{6!}(\frac{\partial^6 v}{\partial x^6} + \frac{\partial^6 v}{\partial y^6}) + ...$

is the truncation error. Expression (1) can be generalized by taking the integral along the circle of radius r around p_0 of the Taylor expansion. Defining $x = rcos(\theta)$ and $y = rsin(\theta)$ as in
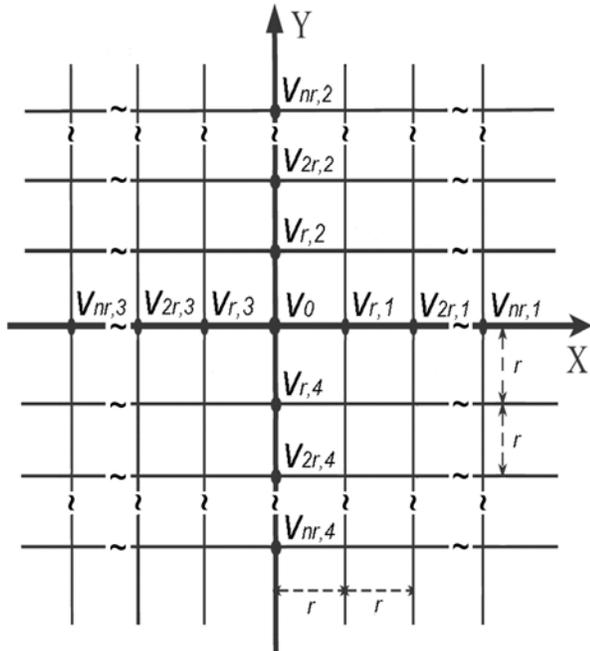
Figure 2. Regular plane square grid with interpoint distance equal to r.

Huiskamp [23] we have:

$$\frac{1}{2\pi} \int_0^{2\pi} v(r,\theta)d\theta - v_0 = \frac{r^2}{4}\Delta v_0$$

$$+ \frac{r^4}{4!} \int_0^{2\pi} \sum_{j=0}^{4} \sin^{4-j}(\theta)\cos^j(\theta)d\theta(\frac{\partial^4 v}{\partial x^{4-j}\partial y^j}) + ... \qquad (2)$$

where $\frac{1}{2\pi} \int_0^{2\pi} v(r,\theta)d\theta$ is the average potential on the ring of radius r and v_0 is the potential on the disc of the CRE.

B. Nine-Point Method (TCRE)

To derive the Laplacian for the TCRE using NPM we add another FPM applying the integral along a circle of radius $2r$ (v_0, $v_{2r,1}$, $v_{2r,2}$, $v_{2r,3}$ and $v_{2r,4}$ on Fig. 2) around point p_0. The following is obtained for the average potential on the ring of radius $2r$ and disc:

$$\frac{1}{2\pi} \int_0^{2\pi} v(2r,\theta)d\theta - v_0 = \frac{(2r)^2}{4}\Delta v_0$$

$$+ \frac{(2r)^4}{4!} \int_0^{2\pi} \sum_{j=0}^{4} \sin^{4-j}(\theta)\cos^j(\theta)d\theta(\frac{\partial^4 v}{\partial x^{4-j}\partial y^j}) + ... \qquad (3)$$

Next, we multiply (2) by 16 and subtract (3) canceling the fourth-order truncation term and resulting in the Laplacian estimation:

$$\Delta v_0 \cong \frac{1}{3r^2}\{16(\frac{1}{2\pi} \int_0^{2\pi} v(r,\theta)d\theta - v_0)$$

$$-(\frac{1}{2\pi} \int_0^{2\pi} v(2r,\theta)d\theta - v_0)\} \qquad (4)$$

III. MAIN RESULTS

A. General (4n + 1)-Point Method for (n + 1)-polar CRE with n Rings

Generalizing (2) and (3) for a case of CRE with n rings ($n \geq 2$) we obtain a set of n FPM equations, one for each ring with radii ranging from r to nr (v_0, $v_{nr,1}$, $v_{nr,2}$, $v_{nr,3}$ and $v_{nr,4}$ on Fig. 2) around point p_0 for which we have:

$$\frac{1}{2\pi} \int_0^{2\pi} v(nr,\theta)d\theta - v_0 = \frac{(nr)^2}{4}\Delta v_0$$

$$+ \frac{(nr)^4}{4!} \int_0^{2\pi} \sum_{j=0}^{4} \sin^{4-j}(\theta)\cos^j(\theta)d\theta(\frac{\partial^4 v}{\partial x^{4-j}\partial y^j}) \qquad (5)$$

$$+ \frac{(nr)^6}{6!} \int_0^{2\pi} \sum_{j=0}^{6} \sin^{6-j}(\theta)\cos^j(\theta)d\theta(\frac{\partial^6 v}{\partial x^{6-j}\partial y^j}) + ...$$

To estimate the Laplacian for this general case the n equations have to be combined in a way that cancels all the truncation terms up to the highest order that can be achieved

for n rings increasing the accuracy of the Laplacian estimation. In order to find such a combination we arrange the coefficients l^k of the truncation terms with the general form $\dfrac{(lr)^k}{k!}\int_0^{2\pi}\sum_{j=0}^{k}\sin^{k-j}(\theta)\cos^j(\theta)d\theta(\dfrac{\partial^k v}{\partial x^{k-j}\partial y^j})$ for order k ranging in increments of 2 from 4 to some even positive integer m ($m \geq 4$) and ring radius multiplier l ranging from 1 [equation (2)] to n [equation (5)] into the (m - 2)/2 by n matrix A as follows:

$$A = \begin{pmatrix} 1^4 & 2^4 & \cdots & n^4 \\ 1^6 & 2^6 & \cdots & n^6 \\ \vdots & \vdots & \ddots & \vdots \\ 1^m & 2^m & \cdots & n^m \end{pmatrix} = \begin{pmatrix} 1 & 2^4 & \cdots & n^4 \\ 1 & 2^6 & \cdots & n^6 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 2^m & \cdots & n^m \end{pmatrix} \quad (6)$$

A matrix equation of the form:

$$A\bar{x} = \bar{0} \quad (7)$$

is equivalent to a homogeneous system of linear equations where $\bar{0}$ is the (m - 2)/2-dimensional zero vector and $\bar{x}$ is the n-dimensional vector that allows the cancellation of all the truncation terms up to the order of m by setting the linear combination of n coefficients l^k corresponding to all ring radii for each order k equal to 0.

The existence of nontrivial solution ($\bar{x} \neq \bar{0}$) of equation (7) depends on the relationship between the number of rows (m - 2)/2 and the number of columns n of matrix A. It is known that for homogeneous systems nontrivial solutions exist only when the system is underdetermined, i.e. (m - 2)/2 < n [24]. Moreover, if A is real as in our case, a real nontrivial solution exists. The largest number of rows the matrix A from (6) may have to stay underdetermined is n - 1, so in order to find the highest truncation term order m that can be cancelled with n rings CREs we solve (m - 2)/2 = n - 1 which yields m = 2n. Therefore, matrix A can be rewritten as an n - 1 by n matrix A' that is a function only of the number of the rings n:

$$A' = \begin{pmatrix} 1 & 2^4 & \cdots & n^4 \\ 1 & 2^6 & \cdots & n^6 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 2^{2n} & \cdots & n^{2n} \end{pmatrix} \quad (8)$$

Equivalently to substituting A' for A into (7) it can be observed that the same nontrivial solutions are given by the null space (or kernel) of matrix A' [24].

B. Examples

In this section (8) is used to numerically estimate the Laplacian for the cases of TCRE (2 rings) and QCRE (3 rings).

For the case of TCRE ($n = 2$) (8) becomes:

$$A' = \begin{pmatrix} 1 & 2^4 \end{pmatrix} \quad (9)$$

One of the integer vectors of the null space of matrix A' from (9) is [16, -1] that was used to estimate the Laplacian for TCRE in (4) as well as in [13]-[22] and other works utilizing TCRE.

For the case of QCRE ($n = 3$) (8) becomes:

$$A' = \begin{pmatrix} 1 & 2^4 & 3^4 \\ 1 & 2^6 & 3^6 \end{pmatrix} \quad (10)$$

One of the integer vectors of the null space of matrix A' from (10) is [270, -27, 2], so the Laplacian estimation that cancels the truncation terms in (5) up to the sixth order for QCRE can be written as follows:

$$\Delta v_0 \cong \frac{1}{45r^2}\{270(\frac{1}{2\pi}\int_0^{2\pi}v(r,\theta)d\theta - v_0)$$

$$-27(\frac{1}{2\pi}\int_0^{2\pi}v(2r,\theta)d\theta - v_0) \quad (11)$$

$$+2(\frac{1}{2\pi}\int_0^{2\pi}v(3r,\theta)d\theta - v_0)\}$$

It should be noted that null space vectors used for TCRE and QCRE Laplacian estimations in (4) and (11) respectively are not unique. From the properties of matrix multiplication it is known that for any vector $\bar{x}$ that belongs to the null space of matrix A and a scalar c the scaled vector $c\bar{x}$ also belongs to the null space of the same matrix A since $(cA)\bar{x} = c(A\bar{x})$. Therefore, any scaled version of given null space vector would also be a null space vector.

Estimation of the Laplacian for any other number of rings $n \geq 2$ can be performed in the identical manner.

IV. DISCUSSION

In this study we propose the general approach to estimating the Laplacian for (n + 1)-polar CREs using the ($4n$ + 1)-point method for $n \geq 2$. This approach allows cancelling all the truncation terms up to the order of $2n$ which is demonstrated to be the highest order achievable for a CRE with n rings. It should be noted that the accuracy of Laplacian estimation increases with the increase of n due to elimination of higher order truncation terms. The general approach is illustrated with two examples deriving the currently used Laplacian estimation for TCRE and, for the first time, introducing the Laplacian estimation for QCRE.

The directions of future work are two-fold. First, for any $n \geq 2$ the null space vectors of matrix A' in (8) can easily be calculated numerically through finding the column echelon form of the matrix using methods like Bareiss algorithm which for exactly given integer matrices such as A' in (8) have been shown to be more efficient than the standard Gaussian elimination [25]. However, deriving the analytic expression for the null space vectors of A' as a function of n would be even more efficient in terms of its computation. Second, while the accuracy of Laplacian estimation increases with the increase of n, computer simulations need to be

performed to assess how significant this gain in accuracy is for practical applications. We plan to perform a comparison between Laplacian estimations for bipolar CRE, TCRE, QCRE and higher order multipolar CREs using both basic single dipole and advanced concentric sphere head models.

V. CONCLUSION

With TCREs gaining increased recognition in a wide range of applications due to their unique capabilities this study lays the groundwork for higher order multipolar CREs. We demonstrate that such higher order multipolar CREs have the potential to offer more accurate Laplacian estimation further suggesting the superiority of CREs as an alternative to conventional disc electrodes for applications not just limited to EEG but with potential for electrocardiography, electromyography, etc.

REFERENCES

[1] J. E. Desmedt, V. Chalklin and C. Tomberg, "Emulation of somatosensory evoked potential (SEP) components with the 3-shell head model and the problem of 'ghost potential fields' when using an average reference in brain mapping," *Electroenceph. Clin. Neurophysiol.*, vol. 77, pp. 243-258, 1990.

[2] P.L. Nunez, R.B. Silberstein, P.J. Cadiush, J.Wijesinghe, A.F.Westdorp, and R. Srinivasan, "A theoretical and experimental study of high resolution EEG based on surface laplacians and cortical imaging," *EEG and Clin. Neurophysiology*, vol. 90, pp. 40-57, 1994.

[3] G. Lantz, R. Grave de Peralta, L. Spinelli, M. Seeck, and C. Michel, "Epileptic source localization with high density EEG: how many electrodes are needed?" *Clin. Neurophys.*, vol. 114, pp. 63-69, 2003.

[4] R. Srinivasan, "Methods to Improve the Spatial Resolution of EEG," *J. Bioelectromagnetism*, vol. 1, pp. 102-111, 1999.

[5] B. He, "Brain Electrical Source Imaging: Scalp Laplacian Mapping and Cortical Imaging," *Crit Rev. Biomed Eng*, vol. 27, pp. 149-188, 1999.

[6] B. He, J. Lian, and G. Li, "High-resolution EEG: a new realistic geometry spline Laplacian estimation technique," *Clin. Neurophysiol.*, vol. 112, pp. 845-852, 2001.

[7] F. Perrin, O. Bertrand, and J. Pernier, "Scalp Current Density Mapping: Value and Estimation from Potential Data," *IEEE Trans Biomed Eng*, vol. 34, pp. 283-288, 1987.

[8] S.K. Law, P.L Nunez, R.S. Wijesinghe, "High resolution EEG using Spline Generated Surface Laplacians on Spherical and Ellipsoidal Surfaces," *IEEE Trans Biomed Eng*, vol. 40, pp.145-153, 1993.

[9] F. Babiloni, C. Babiloni, F. Carducci, L. Fattorini, P. Onorati, and A. Urbano, " Spline Laplacian estimate of EEG potentials over a realistic magnet resonance constructed scalp surface model," *EEG and Clin. Neurophysiology*, vol. 98, pp 363-373, 1996.

[10] D. Farino, and C. Cescon, "Concentric-Ring Electrode systems for Noninvasive detection of single motor unit activity," *IEEE Trans Biomed Eng*, vol. 48, pp. 1326-1334, 2001.

[11] C. D. Klug, J. Silny, and G. Rau, "Improvement of Spatial Resolution in Surface EMG: A Theoretical and Experimental Comparison of Different Spatial Filters," IEEE Trans Biomed Eng, vol. 44(7), pp. 567-574, 1997.

[12] A. Van Oosterom and J. Strackee, "Computing the lead field of electrodes with axial symmetry," *Med. Biol. Eng. Comput.*, vol. 21, pp. 473–481, 1983.

[13] W. Besio, W. Aakula, K. Koka, and W. Dai, "Development of a tri-polar concentric ring electrode for acquiring accurate Laplacian body surface potentials," *Ann Biomed Eng*, vol. 34, pp. 226-235, 2006.

[14] W. Besio, K. Koka, W. Aakula, and W. Dai, "Tri-polar concentric ring electrode development for Laplacian electroencephalography," *IEEE Trans Biomed Eng*, vol. 53, pp. 926-933, 2006.

[15] K. Koka and W. Besio, "Improvement of spatial selectivity and decrease of mutual information of tri-polar concentric ring electrodes," *J of Neuroscience Methods*, vol. 165, pp. 216-222, 2007.

[16] W. Besio, H. Cao, P. Zhou, "Application of tripolar concentric electrodes and pre-feature selection algorithm for brain-computer interface," *IEEE Trans Neural Systems & Rehab Eng*, vol. 16, pp. 191-194, 2008.

[17] O. Makeyev, X. Liu, H. Luna-Munguía, G. Rogel-Salazar, S. Mucio-Ramirez, Y. Liu, Y. Sun, S. Kay, W. Besio, "Toward a noninvasive automatic seizure control system in rats with transcranial focal stimulations via tripolar concentric ring electrodes," *IEEE Trans Neural Systems & Rehab Eng*, vol. 20, pp. 422-431, July 2012.

[18] A. Feltane, G.F. Boudreaux-Bartels, W. Besio, "Automatic seizure detection in rats using Laplacian EEG and verification with human seizure signals," *Annals of Biomedical Engineering*, DOI: 10.1007/s10439-012-0675-4.

[19] W. Besio, K. Koka, and A. Cole, "Effects of noninvasive transcutaneous electrical stimulation via concentric ring electrodes on pilocarpine-induced status epilepticus in rats," *Epilepsia*, vol. 48, pp. 2273-2279, December 2007.

[20] W. Besio, X. Liu, L. Wang, A. Medvedev, and K. Koka, "Transcutaneous electrical stimulation via concentric ring electrodes reduced pentylenetetrazole-induced synchrony in beta and gamma bands in rats," *International Journal of Neural Systems*, vol. 21, pp. 139-149, April 2011.

[21] O. Makeyev, H. Luna-Munguía, G. Rogel-Salazar, X. Liu, W. Besio, "Noninvasive transcranial focal stimulation via tripolar concentric ring electrodes lessens behavioral seizure activity of recurrent pentylenetetrazole administrations in rats," *IEEE Trans Neural Systems & Rehab Eng*, DOI: 10.1109/TNSRE.2012.2198244.

[22] W. Besio, O. Makeyev, A. Medvedev, K. Gale, "Effects of transcranial focal stimulation via tripolar concentric ring electrodes on pentylenetetrazole-induced seizures in rats," *Epilepsy Research*, DOI: 10.1016/j.eplepsyres.2012.12.002.

[23] G. Huiskamp, "Difference formulas for the surface Laplacian on a triangulated surface," *Journal of Computational Physics*, vol. 95, pp. 477-496, 1991.

[24] F. Szabo, *Linear Algebra: An Introduction Using Mathematica*. San Diego: Academic Press, 2000.

[25] E. Bareiss, "Sylvester's identity and multistep integer-preserving Gaussian elimination", *Mathematics of Computation*, vol. 22, pp. 565-578, 1968.