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Comparison of Bipolar vs. Tripolar Concentric Ring Electrode Laplacian Estimates

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Abstract— **Potentials on the body surface from the heart are of a spatial and temporal function. The 12-lead electrocardiogram (ECG) provides useful global temporal assessment, but it yields limited spatial information due to the smoothing effect caused by the volume conductor. The smoothing complicates identification of multiple simultaneous bioelectrical events.**

In an attempt to circumvent the smoothing problem, some researchers used a five-point method (FPM) to numerically estimate the analytical solution of the Laplacian with an array of monopolar electrodes. The FPM is generalized to develop a Bi-polar concentric ring electrode system.

We have developed a new Laplacian ECG sensor, a Tri-electrode sensor, based on a nine-point method (NPM) numerical approximation of the analytical Laplacian. For a comparison, the NPM, FPM and compact NPM were calculated over a 400 x 400 mesh with 1/400 spacing. Tri and Bi-electrode sensors were also simulated and their Laplacian estimates were compared against the analytical Laplacian.

We found that Tri-electrode sensors have a much-improved accuracy with significantly less Relative and Maximum errors in estimating the Laplacian operator. Apart from the higher accuracy, our new electrode configuration will allow better localization of the electrical activity of the heart than Bi-electrode configurations.

Keywords—**Laplacian, nine point method, Relative error, Maximum error, ECG, EEG, electroencephalography**

I. INTRODUCTION

Body surface potential mapping (BSPM) is a method for improving the spatial resolution of electroencephalography. They can be obtained by recording from a large number of electrodes on the body surface unlike the 12-lead electrode systems. Since BSPMs utilize potentials they also are prone to limited spatial resolution due to the smoothing effect of the volume conductor. The Laplacian of the surface potentials, the second spatial derivative of the surface potentials, may help to sharpen the potentials that were smoothed by the volume conductor effect. In recent studies it was found that body surface Laplacian mapping (BSLM) achieved better spatial resolution in localizing and resolving multiple simultaneously active regional cardiac electrical activity than potential mapping [1].

Fattorusso and Tilmant [2] first reported using the concentric ring electrodes in cardiology. Recently He and Cohen [3] developed a concentric ring Bi-electrode sensor, based on a Five-Point numerical approximation to the analytical solution of Laplacian operator. He and Cohen [4] reported that this special Bi-polar surface Laplacian sensor produced higher resolution maps than BSPMs.

This paper will discuss a new Tri-electrode method for measuring surface Laplacian ECG (LECG) that achieves much higher accuracy in duplicating the analytical Laplacian, improved spatial resolution and localization over Bi-polar disc and ring electrode systems. This sensor is based on a numerical approximation technique, the "Nine Point Method" (NPM) which is commonly used in image processing for edge detection.

II. BACKGROUND THEORY

1) *Five Point Method(FPM)*: In the Fig. 1 the Laplacian Δ at point p_0 due to the potentials v_5, v_6, v_7, v_8 and v_0 with spacing of $2r$ [5] is given by

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=\Delta v_0=\frac{1}{(2r)^2}\left(\sum_{i=1}^{4} v_i - 4v_0\right)+O(2r)^2 \quad (1)$$

Where

$$O(r^2)=\frac{(2r)^2}{4!}\left(\frac{\partial^4 v}{\partial x^4}+\frac{\partial^4 v}{\partial y^4}\right)+\frac{(2r)^4}{4!}\left(\frac{\partial^6 v}{\partial x^6}+\frac{\partial^6 v}{\partial y^6}\right)+\cdots$$

is truncation error. This is neglected in [4].

The approximation to the Laplacian at p_0 is then

$$\Delta v_0 \cong \frac{4}{(2r)^2}\left(\bar{v}-v_0\right) \quad (2)$$

where $\bar{v}=\frac{1}{4}\sum_{i=5}^{8} v_i$ is the average of the potentials on four points.

The above equation can be applied to a concentric disc and ring Bi-electrode sensor by taking the integral along the circle of radius r around the point p_0 and defining $x= r\cos(\theta)$ $y= r\sin(\theta)$ [5]. Then (2) becomes

$$\Delta v_0 \cong \frac{4}{(2r)^2}\frac{1}{2\pi}\int_0^{2\pi}\left(v(r,\theta)-v_0\right)d\theta \quad (3)$$

$$\Delta v_0 \cong \frac{4}{(2r)^2}\left(\bar{v}-v_0\right) \quad (4)$$

where $\bar{v}=\frac{1}{2\pi}\int_0^{2\pi} v(r,\theta)d\theta$ which is the average potential on the outer ring.

2) *Compact Nine Point Method(CNPM)*: The Nine-Point arrangement as shown in Fig. 1 can be interpreted as two FPMs. Points v_1, v_2, v_3, v_4 and v_0 forming one FPM with a spacing of *r* and the diagonal points $v_9, v_{10}, v_{11}, v_{12}$ and v_0 with a spacing of *r* forming the second FPM. The Laplacian of the potential at p_0 [6] due to these potentials is given by

$$\Delta v_0 = \frac{1}{6r^2}\left[4\sum_{i=1}^{4} v_i + \sum_{j=5}^{8} v_j - 20v_0\right] + O(r^2)\,. \qquad (5)$$

3) *NPM*: The Nine-Point arrangement as shown in Fig. 1 can be seen as two FPMs. Points v_1, v_2, v_3, v_4 and v_0 forming one FPM with a spacing of *r* and points v_5, v_6, v_7, v_8 and v_0 forming a second FPM with spacing of *2r*. The Laplacian of the potential at point p_0 [6] due to these potentials is

$$\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) = \Delta v_0 = \frac{1}{12r^2}\left\{16\sum_{i=1}^{4} v_i - 60v_0 - \sum_{i=5}^{8} v_i\right\} + O(r^4) \quad (6)$$

where $O(r^4) = \dfrac{r^4}{270}\left(\dfrac{\partial^6 v}{\partial x^6} + \dfrac{\partial^6 v}{\partial y^6}\right) + \cdots$ is the truncation error.

Comparing (1) and (5), it can be observed that the NPM truncation error does not have the 4th order derivative term. Therefore the NPM is more accurate than the FPM and also the CNPM.

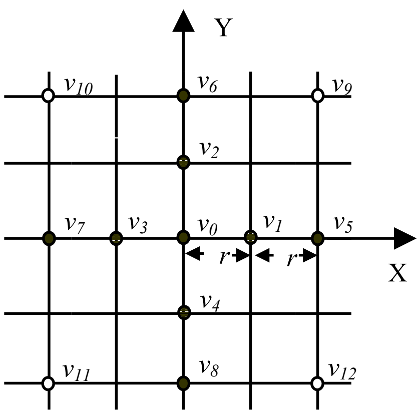


Fig. 1: Arrangement of FPM, CNPM and NPM on a regular plane square grid of size N x N and spacing h =1/N. Inter-point distance r = nh where n=1,2,3,… v_0 to v_{12} are the potentials at points p_0 to p_{12} respectively. v_5, v_6, v_7, v_8 and v_0 form the FPM , v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8 and v_0 forming NPM and v_1, v_2, v_3, v_4, $v_9, v_{10}, v_{11}, v_{12}$ and v_0 forming the CNPM.

III. Methodology

1) *Applying NPM to the New Tri-electrode sensor*: As explained before this NPM can be treated as two FPMs. Then by applying a similar procedure as used for the Bi-electrode sensor, we take the integral along a circle of radius *r* around point p_0 and defining X= rsin(θ), Y= rcos(θ)[5] we get

$$\left(\int_0^{2\pi}(v(r,\theta)d\theta - v_0)d\theta\right) = \frac{r^2}{4}2\pi\Delta v_0$$
$$+\frac{r^4}{24}\int_0^{2\pi}\sum_{j=0}^{4}(\sin\theta)^{4-j}(\cos\theta)^j\left(\frac{\partial^4 v}{\partial x^{4-j}\partial y^j}\right) + \cdots \quad (7)$$

Similarly taking the integral along a circle of radius *2r* around p_0 results in

$$\int_0^{2\pi}\left(v(2r,\theta) - v_0\right)d\theta = r^2 2\pi\Delta v_0$$
$$+\frac{2r^4}{3}\int_0^{2\pi}\sum_{j=0}^{4}(\sin\theta)^{4-j}(\cos\theta)^j\left(\frac{\partial^4 v}{\partial x^{4-j}\partial y^j}\right)$$
$$+\frac{(2r)^6}{6!}\int_0^{2\pi}\sum_{j=0}^{6}(\sin\theta)^{6-j}(\cos\theta)^j\left(\frac{\partial^6 v}{\partial x^{6-j}\partial y^j}\right) + \cdots \qquad (8)$$

Combining equations (7) & (8) as {16*(7)-(8)} so as to cancel out the fourth order term, then the Laplacian approximation becomes

$$\Delta v_0 \cong \frac{1}{3r^2}\left\{\begin{array}{l}16\left(\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} v(r,\theta)d\theta - v_0\right) \\ -\left(\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} v(2r,\theta)d\theta - v_0\right)\end{array}\right\} \qquad (9)$$

where $\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} v(r,\theta)d\theta$ represents the average potential on the middle ring and $\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} v(2r,\theta)d\theta$ represents the average potential on the outer ring.

2) *Comparing Laplacian approximations of NPM, FPM & CNPM:* A mesh of 400 x 400 is constructed with a spacing of 1/400. On each and every point of this mesh, both the FPM and NPMs are applied to approximate the Laplacian. This process is repeated for different r values as explained in Fig.1. These estimates are then compared with the calculated analytical Laplacian for each point of the mesh. To calculate the analytical Laplacian, first the electric potentials generated by a dipole in a homogeneous medium of conductivity σ [1] were calculated using the formula for the electrical potential in (10).

$$\phi = \frac{1}{4\pi\sigma} \cdot \frac{(\overline{r_p} - \overline{r}) \cdot \overline{P}}{\left|\overline{r_p} - \overline{r}\right|^3} \quad (10)$$

Where $\overline{r}$ =(x, y, z) and $\overline{P}$ =(p_x, p_y, p_z) represent the location and moment of the dipole, and $\overline{r_p}$ =(x_p, y_p, z_p) represents the observation point. Then the analytical Laplacian was found by taking the second derivative of the potential as below

$$L = \Delta\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}$$

$$L = \frac{3}{4\pi\sigma}\left[5(z_p - z)^2 \frac{(\overline{r_p} - \overline{r}) \cdot \overline{P}}{\left|\overline{r_p} - \overline{r}\right|^7} - \frac{(\overline{r_p} - \overline{r}) \cdot \overline{P} + 2(z_p - z)p_z}{\left|\overline{r_p} - \overline{r}\right|^5}\right]. \quad (11)$$

The Laplacian approximations of the two finite difference methods are compared with those of the analytical by calculating the *Relative Error* and *Maximum Error* [7].

$$\text{RELERR}^i = \left[\frac{\sum(\Delta v - \Delta^i v)^2}{\sum(\Delta v)^2}\right]^{\frac{1}{2}} \quad (12)$$

$$\text{MAXERR}^i = \max\left|\Delta v - \Delta^i v\right| \quad (13)$$

where i represents the method used to find the Laplacian and Δv represents the analytical Laplacian of the potential.

3) *Comparing Laplacian approximation of Bi-electrode and Tri-electrode sensor*: The Tri-electrode sensor as shown in Fig. 2 can be treated as either a Bi-electrode or a Tri-electrode sensor by ignoring (utilizing) the dashed ring for Bi-electrode (Tri-electrode) calculations. This Tri-electrode sensor is simulated at a particular height above the origin as shown in Fig. 3 by taking 0.01 times 2*pi as the angular displacement between each point on the rings. A dipole is simulated which is oriented to the surface in the positive direction of the Z-axis as shown in Fig. 3.

Using this Tri & Bi-electrode configuration, the Laplacian is calculated at the center with varying inter-electrode distance (n) and with a fixed dipole at the origin. These Laplacian values are compared with the analytical values by calculating the *Relative Error* and *Maximum Error* [7].

IV. RESULTS

A. Error calculation for the NPM, FPM & CNPM

Relative and Maximum errors of the NPM, FPM and CNPM are calculated using formulae (12) and (13) when compared to the analytical value of Laplacian calculated using (11). The errors are plotted for different inter point distances (r) as shown in Fig. 4.

B. Error calculation for Bi & Tri electrode sensors

Relative error and Maximum errors of Bi-electrode and Tri-electrode sensors are tabulated in Table I. Two sample t-tests which assumed unequal variance were used to test significance. The Relative errors of the NPM were compared both between the FPM and the CNPM. The Maximum errors of the NPM were compared similarly. It was found that the NPM was significantly better in all four cases at the 1% confidence level. The mean percent improvement of error by the NPM compared to the other methods ranged from 99.65% to 99.88%.

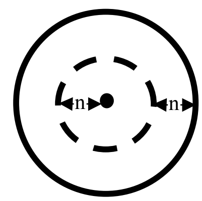


Fig. 2: Tri-electrode sensor, n is inter-electrode distance

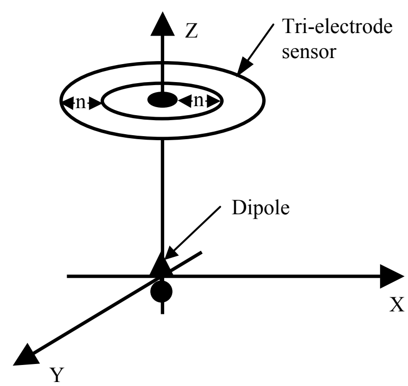
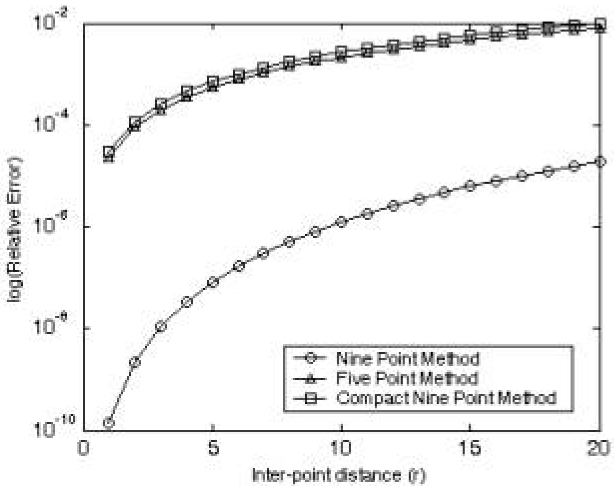


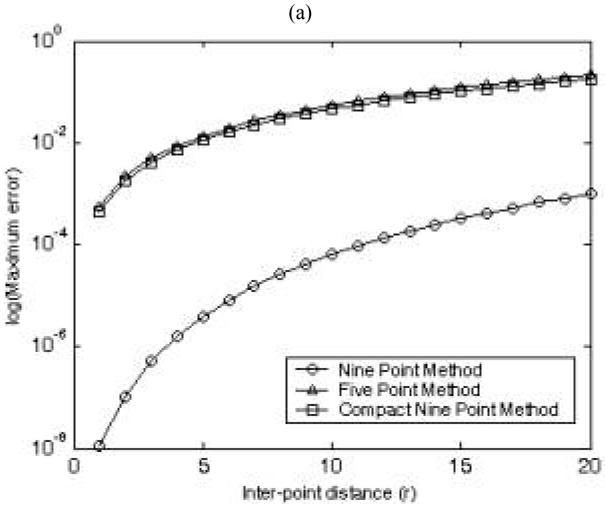
Fig. 3: Schematic of simulation used for Bi-electrode and Tri-electrode sensors at a particular height above the origin with fixed dipole at the origin.

Table I
Relative and Maximum errors for Bi and Tri-electrode sensors with various radii

Radius n - units	Bi-electrode Sensor		Tri-electrode Sensor	
	Rel. error	Max. error	Rel. error	Max. error
0.5	1.91813e-2	4.49563e-4	8.70885e-5	2.04113e-6
1	7.28031e-2	1.70632e-3	1.30743e-3	3.06431e-5
1.5	1.50892e-1	3.53653e-3	5.98458e-3	1.40263e-4
2	2.41444e-1	5.65885e-3	1.65892e-2	3.88811e-4
2.5	3.3405e-1	7.82936e-3	3.47052e-2	8.13403e-4
3	4.21629e-1	9.88194e-3	6.06464e-2	1.42140e-3



(a)



(b)

Fig. 4: Relative Error (a) Maximum Error (b) of NPM, FPM and Compact NPM when compared to the analytical value of the Laplacian

V. DISCUSSIONS

The results of the simulations for the Maximum and Relative error of the NPM, FPM and CNPM are plotted on a semi-log graph as shown in Fig. 4 and Bi-electrode and Tri-electrode data are tabulated in Table 1. The FPM and CNPM both have truncation error on the order of r^2 where as the NPM has order of r^4. Hence, we expected the NPM to be more accurate than FPM and CNPM, which is proven by our simulations. The two sample t-tests results showed there was a significant difference in error approximating the Laplacian between the NPM and both of the other finite difference methods compared and between Tri and Bi-electrode configurations as well. The NPM is generalized to approximate the Laplacian for the Tri-electrode sensor. We expected the Tri-electrode sensor to be more accurate than the Bi-electrode sensor due to the difference in truncation error, and it is proven so by the simulation results.

VI. CONCLUSIONS

The Tri-electrode configuration gives a significantly better approximation to the analytical Laplacian than the other finite difference methods commonly used. This should be helpful in localizing sources, which will be our future work. We will also conduct tank experiments to verify our simulation results. With an array of these Tri-electrode sensors, Laplacian surface potential maps should be made more accurately than past mapping efforts.

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