DEVELOPMENT OF TRI-POLAR LAPLACIAN ELECTROCARDIOGRAM ELECTRODES

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Potentials on the body surface from the heart are of a spatial and temporal function. The 12-lead electrocardiogram (ECG) provides useful global temporal assessment, however it yields limited spatial information due to the smoothing effect of the volume conductor.

We have developed a unique Laplacian ECG sensor, a concentric tri-polar electrode system, based upon a nine-point finite difference method approximation of the analytical Laplacian. This gives a more accurate solution of the Laplacian than a concentric bi-polar electrode. Tri-polar and bipolar electrode systems were simulated on a mesh and their Laplacian estimates were compared with the analytical Laplacian.

It was found that the concentric tri-polar electrode system has a much-improved accuracy with less relative and maximum errors in the estimation of the Laplacian operator. Due to the higher accuracy, the improved electrode configuration will allow more precise localization of electrical activity of the heart when compared to the concentric bi-polar configuration.

1. Introduction and Background

In Fig.1 the Laplacian Δ, using Five Point Method (FPM) numerical approximation, at point v_0 due to the potentials v_5,v_6,v_7,v_8 and v_0 with spacing of *2r* is discussed in [1]. The Laplacian Δ, using Compact Nine Point Method (CNPM) numerical approximation, at v_0 due to the potentials v_1, v_2, v_3, v_4, v_9, v_{10}, v_{11}, v_{12} and v_0 is discussed in [2].

1.1. *Nine Point Method (NPM)*

By Ames [2] the Laplacian of the potentials at v_0 due to the Nine-Point arrangement formed by v_1,v_2,v_3,v_4, v_5,v_6,v_7,v_8 and v_0 shown in Fig. 1 is

$$\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)=\Delta v_0=\frac{1}{12r^2}\left\{16\sum_{i=1}^{4}v_i-60v_0-\sum_{i=5}^{8}v_i\right\}+O(r^4) \qquad (1)$$

where $O(r^4)$ is the truncation error.

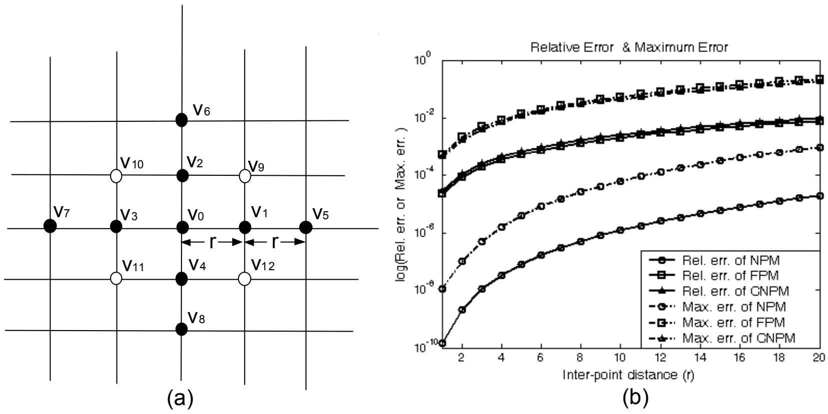

Figure 1. (a) Arrangement of different finite difference methods analyzed. (b) Semi-log graph showing relative error and maximum error of FPM, CNPM & NPM.

2. Methodology

2.1. *Comparing Laplacian Approximations of FPM, CNPM & NPM And Error Calculations*

A computer simulation is performed by simulating a mesh of 400 x 400 with a spacing of 1/400 at a certain height above the origin and a dipole at the origin, which is oriented in positive direction of the Z-axis. First the electric potentials generated by a dipole in a homogeneous medium of conductivity σ were calculated at each and every point using the procedure mentioned by He and Wu [3]. Then on each point of this mesh the FPM, CNPM and NPM were applied to approximate the Laplacian. This process is repeated for different inter-point distances. The analytical Laplacian was calculated for each point of the mesh using the procedure mentioned in [3].

The Laplacian approximations of the three finite difference methods were compared with those of the analytical values by calculating the *Relative Error* and *Maximum Error* [4] for different inter-point distances.

$$\mathrm{RELERR}^{\mathrm{i}} = \left[\frac{\sum \left(\Delta v - \Delta^{i} v \right)^{2}}{\sum \left(\Delta v \right)^{2}} \right]^{\frac{1}{2}} \tag{2}$$

$$\mathrm{MAXERR}^{\mathrm{i}} = \max \left| \Delta v - \Delta^{i} v \right| \tag{3}$$

Where *i* represents the method used to find the Laplacian and Δv represents the analytical Laplacian of the potential. The errors are plotted for different inter point distances (r) on a semi-log graph as shown in Fig. 1.

2.2. *Applying NPM To The Concentric Tri-polar Electrode*

The NPM can be generalized to apply for a concentric tri-polar electrode by taking the integral along a circle of radius r around point p_0 and defining X= rsin(θ), Y= rcos(θ)[4] which results in

$$\left(\int_0^{2\pi}(v(r,\theta)d\theta - v_0)d\theta\right) = \frac{r^2}{4}2\pi\Delta v + \frac{r^4}{24}\int_0^{2\pi}\sum_{j=0}^{4}(\sin\theta)^{4-j}(\cos\theta)^j\left(\frac{\partial^4 v}{\partial x^{4-j}\partial y^j}\right) + \cdots \tag{4}$$

Similarly taking the integral along a circle of radius *2r* around p_0 and defining X= 2rsin(θ), Y=2rcos(θ)[4] results in

$$\int_0^{2\pi}\left(v(2r,\theta)-v_0\right)d\theta = r^2 2\pi\Delta v_0 + \frac{2r^4}{3}\int_0^{2\pi}\sum_{j=0}^{4}(\sin\theta)^{4-j}(\cos\theta)^j\left(\frac{\partial^4 v}{\partial x^{4-j}\partial y^j}\right) + \frac{(2r)^6}{6!}\int_0^{2\pi}\sum_{j=0}^{6}(\sin\theta)^{6-j}(\cos\theta)^j\left(\frac{\partial^6 v}{\partial x^{6-j}\partial y^j}\right) + \cdots \tag{5}$$

Combining equations (4) & (5) as {16*(4)-(5)} cancels out the fourth order term, and the Laplacian approximation becomes

$$\Delta v_0 \cong \frac{1}{3r^2}\left\{16\left(\frac{1}{2\pi}\int_0^{2\pi}v(r,\theta)d\theta - v_0\right) - \left(\frac{1}{2\pi}\int_0^{2\pi}v(2r,\theta)d\theta - v_0\right)\right\} \tag{6}$$

where $\frac{1}{2\pi}\int_0^{2\pi}v(r,\theta)d\theta$ represents the average potential on the middle ring and $\frac{1}{2\pi}\int_0^{2\pi}v(2r,\theta)d\theta$ represents the average potential on the outer ring.

2.3. *Comparing Laplacian Approximations of Bi-polar & Tri-polar Electrodes And Error Calculations.*

The concentric tri-polar electrode is simulated on the same mesh used to compare FPM, CNPM and NPM. This tri-polar electrode can be considered as bi-polar by neglecting the middle ring while calculating the Laplacian. The Laplacian is calculated with varying inter-electrode distance (n). The Laplacian estimates are compared with the analytical values by calculating the *Relative Error* and *Maximum Error* [4]. These values are tabulated in Table I.

3. Discussions & Conclusions

The FPM and CNPM both have truncation error on the order of r^2 where as the NPM has order of r^4. Hence, it was assumed that the NPM and the tri-

Table 1. Relative and Maximum errors for Bi and Tri-polar electrode with various radii.

Radius n - units	Bi-polar electrode		Tri-polar electrode	
	Rel. err.	Max. err.	Rel. err.	Max. err.
0.5	1.91813e-2	4.49563e-4	8.70885e-5	2.04113e-6
1	7.28031e-2	1.70632e-3	1.30743e-3	3.06431e-5
1.5	1.50892e-1	3.53653e-3	5.98458e-3	1.40263e-4
2	2.41444e-1	5.65885e-3	1.65892e-2	3.88811e-4
2.5	3.3405e-1	7.82936e-3	3.47052e-2	8.13403e-4
3	4.21629e-1	9.88194e-3	6.06464e-2	1.42140e-3

electrode would be more accurate, which was supported by our simulations. Two sample t-tests, which assumed unequal variance, were used to test significance. The relative errors of the NPM were compared between both the FPM and the CNPM. The maximum errors of the NPM were compared similarly. It was found that the NPM was significantly better in all four cases at the 1% confidence level. The mean percent improvement of error by the NPM compared to the other methods ranged from 99.65% to 99.88%. The improved Laplacian estimation should be helpful in localizing sources, which will be our future work. We will also conduct tank experiments to verify our simulation results. With an array of these Tri-polar electrodes, Laplacian surface potential maps should be made more accurately than past mapping efforts.

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