New Measures of Quantum Information

Dr. David Kagan[1] in collaboration with Dr. Jacob Barandes[2]

UMass Dartmouth

Abstract

We define new measures of quantum information that describe information flow across quantum channels. We use these quantities to provide novel perspectives on some standard results in quantum information theory.

Classical vs. Quantum Information

Classical information is encoded in bits. A bit can be either `on' (Blue) or `off' (Red).

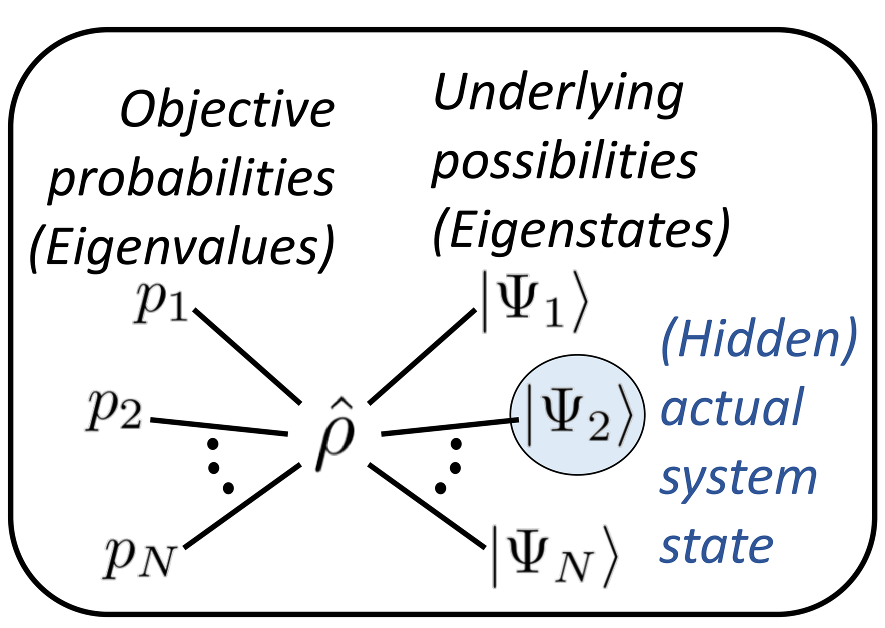
Quantum bits (qubits) can also be `on' ($|\uparrow\rangle$) or `off' ($|\downarrow\rangle$), but **quantum superposition** allows for infinite alternative possible exclusive pairs, such as `right,'

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$$

or `left,'

$$|\leftarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$$

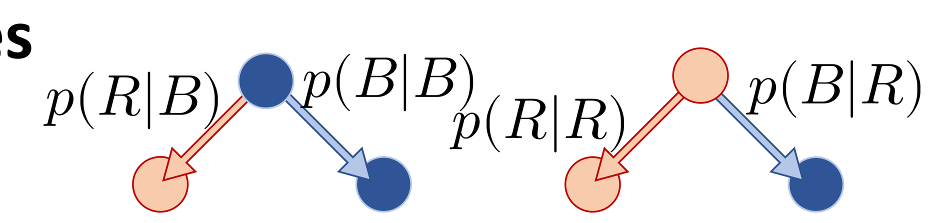
Quantum entanglement hides the underlying state of a system of qubits, forcing a description in terms of an **objective density matrix $\hat{\rho}$.**



The density matrix eigenvectors describe the system's mutually exclusive possible states, while the eigenvalues capture the probabilities that the corresponding eigenvectors represent the *actual* state of the system.

Classical Channels and Conditional Entropy

Conditional probabilities $p(out|in)$ describe noisy classical channels that can randomly change the states of transmitted bits.



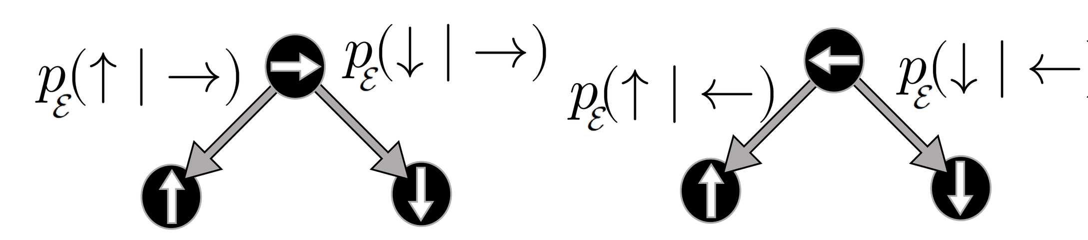
Conditional Shannon entropy

$$H(Y|X) \equiv -\sum_{x,y} p(y|x)p(x)\log[p(y|x)]$$

quantifies information flow. **Mutual information** is a related quantity that describes the information preserved by a channel. The **data processing inequality** is shows that mutual information tends to degrade as information is transmitted across successive classical channels.

Quantum Conditional Probability

A **quantum channel** $\mathcal{E}$ is a linear map from one density matrix to another. Quantum channels define **quantum conditional probabilities** interpretable as the dynamics governing the system's underlying states. The possibilities are richer than the classical case due to the purely quantum evolution of the system density matrix eigenstates.



Concretely, if $|\Psi_q\rangle$ is an initial density matrix eigenstate and $|\Psi_r\rangle$ is an eigenstate of the final density matrix, then

$$p_\mathcal{E}(r|q) \equiv \langle\Psi_r|\mathcal{E}\{|\Psi_q\rangle\langle\Psi_q|\}|\Psi_r\rangle$$

Is the quantum conditional probability that the system occupies underlying state $|\Psi_r\rangle$ given initial state $|\Psi_q\rangle$.

New Quantum Conditional Entropy

Standard quantum conditional entropy does not generically emerge from an underlying probability distribution. **A key result of our work is the definition of new quantum conditional entropy** utilizing quantum conditional probabilities and Shannon's entropy formula, in direct analogy with classical conditional Shannon entropy:

$$J_\mathcal{E}(R|Q) \equiv -\sum_{q,r} p_\mathcal{E}(r|q)p_q\log[p_\mathcal{E}(r|q)]$$

where $\{p_q\}$ are the eigenvalues of the initial density matrix. We also define **new quantum mutual information**

$$I_\mathcal{E}(R:Q) \equiv \sum_{q,r} p_\mathcal{E}(r|q)p_q\log\left[\frac{p_\mathcal{E}(r|q)}{p_r}\right]$$

where $\{p_r\}$ are the eigenvalues of the final density matrix.

Utilizing these new quantum information measures, we are able to prove a **quantum data processing inequality**, finding that **Holevo's chi quantity emerges as a natural bound on the mutual information** shared between a quantum system's initial configuration and its final configuration after ``processing'' by a quantum channel.

Future Directions

New quantum information measures can be used to analyze errors and error correction in noisy quantum systems.

Relations between these quantities, quantum discord, and other information measures is also of interest.

[1]University of Massachusetts Dartmouth
[2]Harvard University

For more details and relevant references use the QR code at the bottom right corner or follow this link to our preprint: https://arxiv.org/abs/2109.07447